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Novel Decomposition Methods for Reliability-Based Topology Optimization

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NOVEL DECOMPOSITION METHODS FOR RELIABILITY-BASED TOPOLOGY OPTIMIZATION

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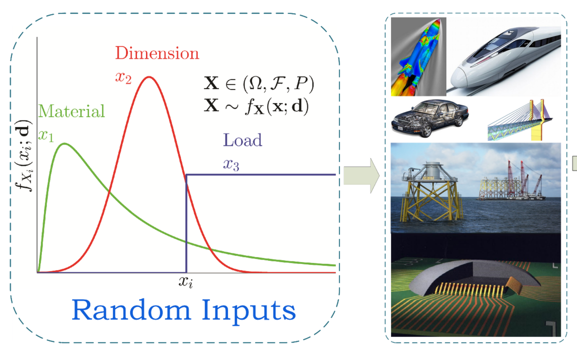
INTRODUCTION



- Moments

$$\mathbb{E}_{\mathbf{d}}[y_l(\mathbf{X})] := \int_{\mathbb{R}^N} y_l(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}; \mathbf{d}) d\mathbf{x}$$

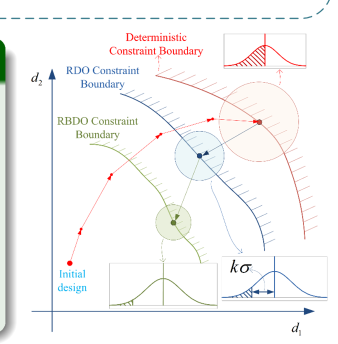
$$\mathrm{var}_{\mathbf{d}}[y_l(\mathbf{X})] := \int_{\mathbb{R}^N} (y_l(\mathbf{x}) - \mathbb{E}_{\mathbf{d}}[y_l(\mathbf{X})])^2 f_{\mathbf{X}}(\mathbf{x}; \mathbf{d}) d\mathbf{x}$$

- Probability of Failure

$$P_{\mathbf{d}}(\mathbf{X} \in \Omega_{F,l}) = \int_{\mathbb{R}^N} I_{\Omega_{F,l}}(\mathbf{x}) f_{\mathbf{X}}(\mathbf{x}; \mathbf{d}) d\mathbf{x}$$

Topology Design under Uncertainty

- Robust Topology Optimization (RTO)
  - Find an optimal topology design with reduced variability of the system performance, leading to insensitive topology design.
- Reliability-Based Topology Optimization (RBTO)
  - Find the optimal topology designs with low probabilities of failure.



INTRODUCTION

- RTO

$$\begin{aligned} \min_{\Omega} \quad & c_0(\Omega) := g_0\left(\mathbb{E}[y_0(\mathbf{X};\Omega)], \mathrm{var}[y_0(\mathbf{X};\Omega)]\right), \\ \text{subject to} \quad & c_l(\Omega) := g_l\left(\mathbb{E}[y_l(\mathbf{X};\Omega)], \mathrm{var}[y_l(\mathbf{X};\Omega)]\right) \le 0, \\ & l = 1, \cdots, K, \\ & \Omega \subseteq D \end{aligned}$$

Needs: moments, design sensitivity of moments, optimization methods

- RBTO

$$\begin{aligned} \min_{\Omega} \quad & c_0(\Omega) := \mathbb{E}[y_0(\mathbf{X};\Omega)], \\ \text{subject to} \quad & c_k(\Omega) := P[\mathbf{X} \in \Omega_{F,k}] \le p_k; \quad k = 1, \cdots, K, \\ & \Omega \subseteq D \end{aligned}$$

Needs: reliability, design sensitivity of reliability, optimization methods

$D \subset \mathbb{R}^3$ is a bounded domain in which all admissible topology design Ω are included

INTRODUCTION

Project Goal

Create new theoretical foundations and numerical algorithms for large-scale, robust topology optimization (RTO) and reliability-based topology optimization (RBTO) of complex engineering systems

Project Objectives

- Develop generalized adaptive-sparse polynomial dimensional decomposition (AS-PDD) for stochastic analysis of general RBTO of high-dimensional systems (Year 1)
- Generate new AS-PDD based formulae and algorithms for moments, reliability, and topology design sensitivities (Year 2)
- Develop fast and reliable optimization algorithms for RTO and RBTO (Year 3)

STOCHASTIC TOPOLOGY OPTIMIZATION

Input $\mathbf{X} \in \mathbb{R}^N$ → COMPLEX SYSTEM → Output $y(\mathbf{X};\Omega) \in \mathcal{L}_2(\mathbb{R})$

$X_i \sim f_{X_i}(x_i)$; indep.; $\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u) = \prod_{p=1}^{|u|} \psi_{i_p j_p}(X_{i_p})$

- Polynomial Dimensional Decomposition (PDD)

$$y(\mathbf{X};\Omega) = y_\emptyset(\Omega) + \sum_{\emptyset \ne u \subseteq \{1,\cdots,N\}} \sum_{\substack{\mathbf{j}_{|u|} \in \mathbb{N}_0^{|u|} \\ j_1,\cdots,j_{|u|} \ne 0}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)$$

- S-variate, mth-order PDD Approximation

$$\tilde{y}_{S,m}(\mathbf{X};\Omega) = y_\emptyset(\Omega) + \sum_{\substack{\emptyset \ne u \subseteq \{1,\cdots,N\} \\ 1 \le |u| \le S}} \sum_{\substack{\mathbf{j}_{|u|} \in \mathbb{N}_0^{|u|}, \|\mathbf{j}_{|u|}\|_\infty \le m \\ j_1,\cdots,j_{|u|} \ne 0}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)$$

$y_\emptyset(\Omega) := \int_{\mathbb{R}^N} y(\mathbf{x};\Omega) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$, $C_{u\mathbf{j}_{|u|}}(\Omega) := \int_{\mathbb{R}^N} y(\mathbf{x};\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{x}_u) f_{\mathbf{X}}(\mathbf{x}) d\mathbf{x}$

STOCHASTIC TOPOLOGY OPTIMIZATION

- Fully AS-PDD Approximation

$$\bar{y}(\mathbf{X};\Omega) := y_\emptyset(\Omega) + \sum_{\emptyset \ne u \subseteq \{1,\cdots,N\}} \sum_{m_u=1}^{\infty} \sum_{\substack{\|\mathbf{j}_{|u|}\|_\infty = m_u,\, j_1,\cdots,j_{|u|} \ne 0 \\ \tilde{G}_{u,m_u} > \epsilon_1, \Delta\tilde{G}_{u,m_u} > \epsilon_2}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)$$

- Partially AS-PDD Approximation

$$\bar{y}_S(\mathbf{X};\Omega) := y_\emptyset(\Omega) + \sum_{\substack{\emptyset \ne u \subseteq \{1,\cdots,N\} \\ 1 \le |u| \le S}} \sum_{m_u=1}^{\infty} \sum_{\substack{\|\mathbf{j}_{|u|}\|_\infty = m_u,\, j_1,\cdots,j_{|u|} \ne 0 \\ \tilde{G}_{u,m_u} > \epsilon_1, \Delta\tilde{G}_{u,m_u} > \epsilon_2}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)$$

Based on Sobol indices:

$$\tilde{G}_{u,m_u} := \frac{1}{\sigma^2(\Omega)} \sum_{\substack{\mathbf{j}_{|u|} \in \mathbb{N}_0^{|u|}, \|\mathbf{j}_{|u|}\|_\infty \le m_u \\ j_1,\cdots,j_{|u|} \ne 0}} C^2_{u\mathbf{j}_{|u|}}(\Omega); \quad \Delta\tilde{G}_{u,m_u} := \frac{\tilde{G}_{u,m_u} - \tilde{G}_{u,m_u-1}}{\tilde{G}_{u,m_u-1}}$$

STOCHASTIC TOPOLOGY OPTIMIZATION

- Two Important Properties of Polynomial Basis

$$\mathbb{E}\left[\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)\right] = 0$$

$$\mathbb{E}\left[\psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u)\psi_{v\mathbf{j}_{|v|}}(\mathbf{X}_v)\right] = \begin{cases} 1 & \text{if } u = v, \\ 0 & \text{if } u \ne v. \end{cases}$$

- Second-Moment Statistics

$$\mathbb{E}[\bar{y}_S(\mathbf{X};\Omega)] = y_\emptyset(\Omega)$$

$$\mathrm{var}[\bar{y}_S(\Omega)] = \sum_{\substack{\emptyset \ne u \subseteq \{1,\cdots,N\} \\ 1 \le |u| \le S}} \sum_{m_u=1}^{\infty} \sum_{\substack{\mathbf{j}_{|u|} \in \mathbb{N}^{|u|}, \|\mathbf{j}_{|u|}\|_\infty = m_u \\ \tilde{G}_{u,m_u} > \epsilon_1, \Delta\tilde{G}_{u,m_u} > \epsilon_2}} C^2_{u\mathbf{j}_{|u|}}(\Omega)$$

STOCHASTIC TOPOLOGY OPTIMIZATION

Given a failure domain $\Omega_F \subset \mathbb{R}^N$ and indicator function I_{Ω_F}, the failure probability for certain topology design Ω

$$P_F(\Omega) := P[\mathbf{X} \in \Omega_F] = \mathbb{E}[I_{\Omega_F}(\mathbf{X})].$$

- Reliability Analysis

$$\bar{P}_{F,S}(\Omega) = \mathbb{E}\left[I_{\bar{\Omega}_{F,S}}(\mathbf{X})\right] = \lim_{L \to \infty} \frac{1}{L} \sum_{l=1}^{L} I_{\bar{\Omega}_{F,S}}(\mathbf{x}^{(l)})$$

$$I_{\bar{\Omega}_{F,S}}(\mathbf{x}^{(l)}) = \begin{cases} 1 & \mathbf{x}^{(l)} \in \bar{\Omega}_{F,S}, \\ 0 & \text{otherwise} \end{cases}$$

$$\bar{\Omega}_{F,S} := \begin{cases} \{\mathbf{x} : \bar{y}_S(\mathbf{x};\Omega) < 0\}, & \text{component}, \\ \{\mathbf{x} : \cup_i \bar{y}_{i,S}(\mathbf{x};\Omega) < 0\}, & \text{series system}, \\ \{\mathbf{x} : \cap_i \bar{y}_{i,S}(\mathbf{x};\Omega) < 0\}, & \text{parallel system}. \end{cases}$$

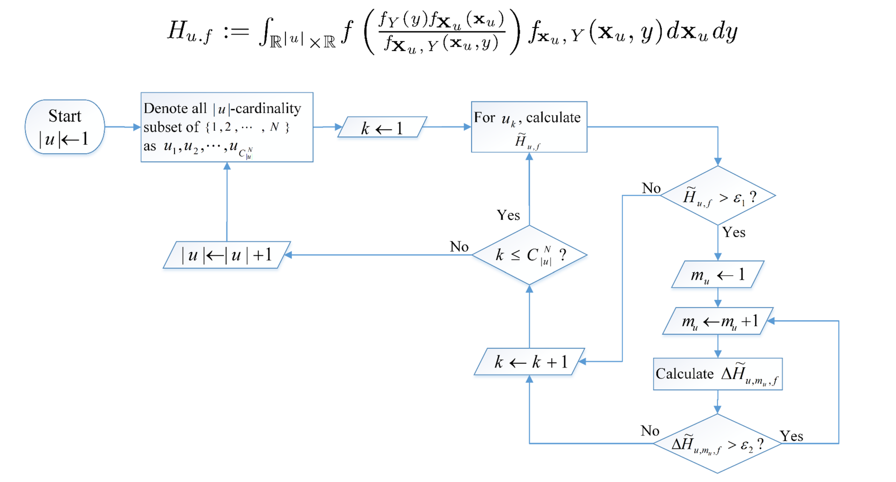
When $S \to N$, $\epsilon_1, \epsilon_2 \to 0$, then $\bar{y}_S \xrightarrow{m.s.} y$, $\bar{y}_{i,S} \xrightarrow{m.s.} y_i$.

STOCHASTIC TOPOLOGY OPTIMIZATION

- New AS-PDD Using f−index

$$\bar{y}(\mathbf{X};\Omega) := y_\emptyset(\Omega) + \sum_{\substack{\emptyset \ne u \subseteq \{1,\cdots,N\} \\ H_{u,f} > \epsilon_1}} \sum_{m_u=1}^{\infty} \sum_{\substack{|\mathbf{j}_{|u|}|_\infty = m_u \\ \Delta H_{u,m_u,f} > \epsilon_2}} C_{u\mathbf{j}_{|u|}}(\Omega) \psi_{u\mathbf{j}_{|u|}}(\mathbf{X}_u),$$

$$H_{u.f} := \int_{\mathbb{R}^{|u|} \times \mathbb{R}} f\left(\frac{f_Y(y) f_{\mathbf{X}_u}(\mathbf{x}_u)}{f_{\mathbf{X}_u,Y}(\mathbf{x}_u, y)}\right) f_{\mathbf{x}_u,Y}(\mathbf{x}_u, y) d\mathbf{x}_u dy$$



STOCHASTIC TOPOLOGY OPTIMIZATION

- Computational Complexity

N = No. of random variables; n = Maximum no. of integration points for single dimension; S = max. degree of interaction

	Reliability	Sensitivity
$S = 1$	$\le nN + 1$	No Additional Fun. Eval.
$S = 2$	$\le n^2 \frac{N(N-1)}{2} + nN + 1$	No Additional Fun. Eval.
$S = 3$	$\le n^3 \frac{N(N-1)(N-2)}{6} + n^2 \frac{N(N-1)}{2} + nN + 1$	No Additional Fun. Eval.

AS-PDD is more efficient than truncated PDD

RESULTS

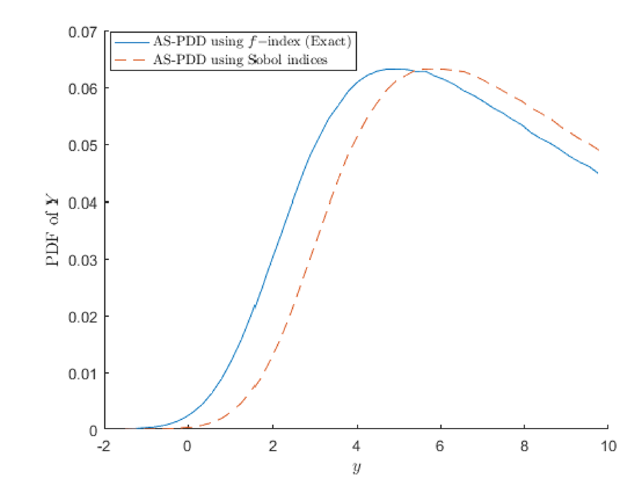
- PDF Estimation of a Mathematical Function

$y(X_1, X_2) = X_1 + 10X_2^2 + 0.01X_2^3$

$X_1 \sim \mathcal{N}(1, 1)$,

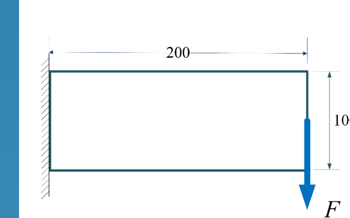
$X_2 \sim \ln\mathcal{N}(0, 1)$

The small tolerance for AS-PDD $\epsilon_2 = 0.005$



RESULTS

- Topology Sensitivity of Moments: a Cantilever Beam

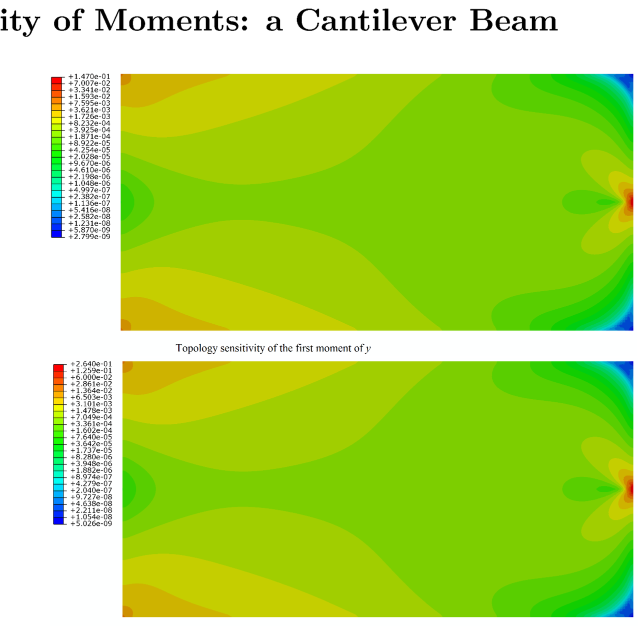


$F \sim \mathcal{N}(2, 1)$

Young's modulus $E \sim \mathcal{N}(100, 1)$

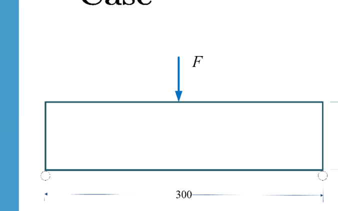
Poisson's ratio $\nu = 0.3$

$y = \int_\Omega \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\epsilon} d\Omega$



RESULTS

- Topology Sensitivity of Moments: a Three Point Bending Case

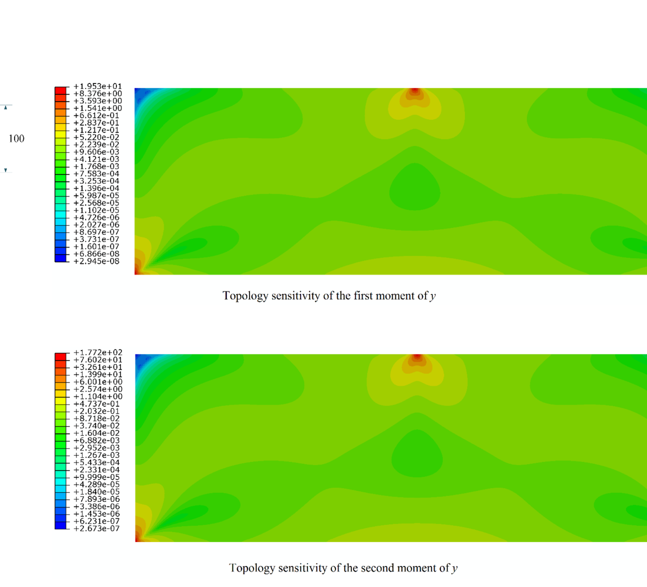


$F \sim \mathcal{N}(10, 1)$

Young's modulus $E \sim \mathcal{N}(100, 1)$

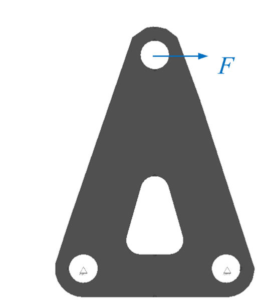
Poisson's ratio $\nu = 0.3$

$y = \int_\Omega \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\epsilon} d\Omega$



RESULTS

- Topology Sensitivity of Moments: a Three-Hole Bracket

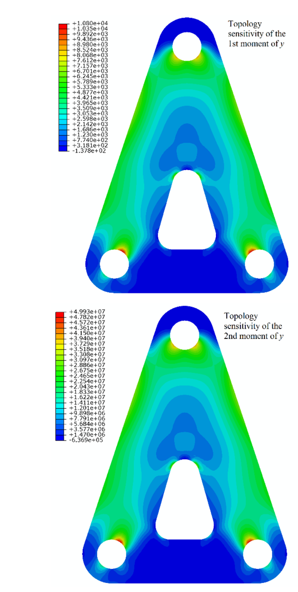


$F \sim \mathcal{N}(15000, 0.1 \times 15000)$

Young's modulus $E \sim \mathcal{N}(207400, 0.1 \times 207400)$

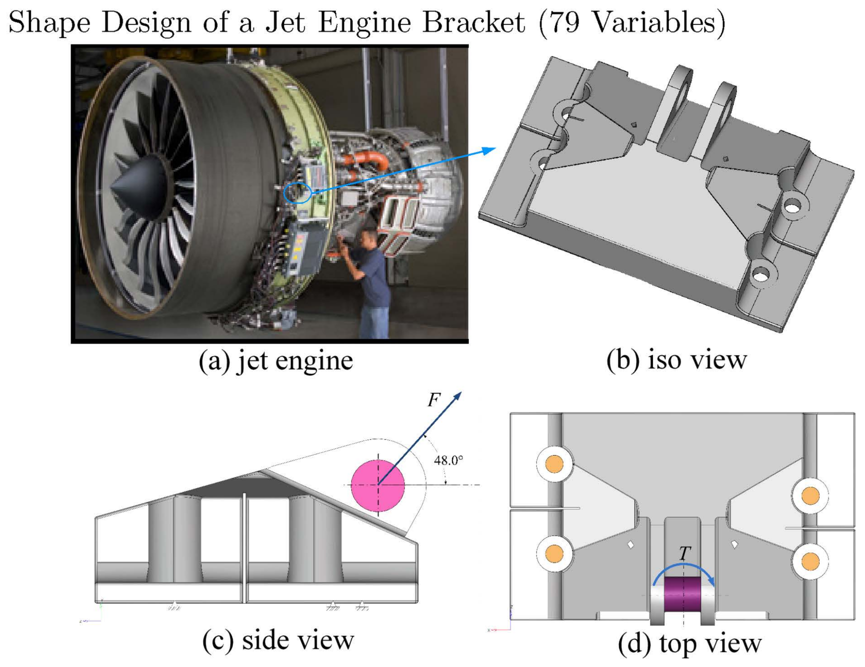
Poisson's ratio $\nu = 0.3$

$y = \int_\Omega \frac{1}{2} \boldsymbol{\sigma} : \boldsymbol{\epsilon} d\Omega$



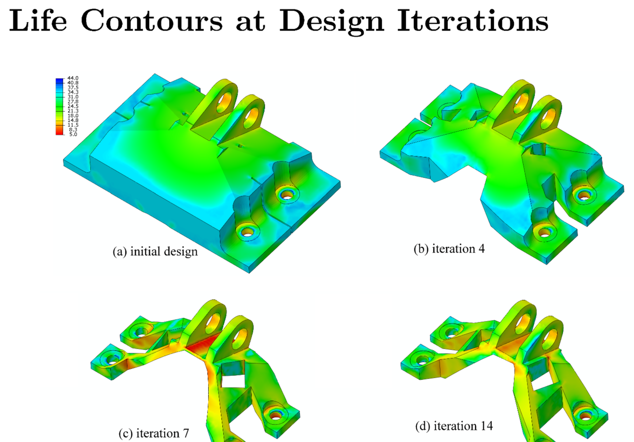
RESULTS

- Shape Design of a Jet Engine Bracket (79 Variables)



RESULTS

- Fatigue Life Contours at Design Iterations



Summary

- "Optimal" mass: 0.4904 kg (84% reduction of initial mass)
- Required 14 iterations and 2808 FEA

PUBLICATIONS & REFERENCES

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